Math 225 (Q1) Homework Assignment 5.

1. Let $y = \begin{pmatrix} 5 \\ -9 \\ 5 \end{pmatrix}$, $u_1 = \begin{pmatrix} 2/3 \\ 1/3 \\ 2/3 \end{pmatrix}$, $u_2 = \begin{pmatrix} -2/3 \\ 2/3 \\ 1/3 \end{pmatrix} \in \mathbf{R}^3$.

   (a) Show that $\{u_1, u_2\}$ is an orthonormal set of vectors.

   (b) Let $W = \text{Span}\{u_1, u_2\}$ be the plane (two dimensional subspace) spanned by the vectors u_1 and u_2. Find $\text{proj}_W(y)$, the projection of the vector y onto the subspace W.

   (c) Find the distance from y to W, that is, find $\|y - \text{proj}_W(y)\|$.

2. Let $A = \begin{pmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{pmatrix}$.

   (a) Find an orthogonal basis for the column space of A.

   (b) Find a QR factorization of A, where Q has orthonormal columns and R is upper triangular matrix with positive diagonal entries.

3. Let $A = \begin{pmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}$.

   (a) Find the orthogonal projection of b onto $\text{Col}(A)$.

   (b) Find a least square solution of $Ax = b$ using part (a).

   (c) Construct the normal equation for $Ax = b$.

   (d) Solve the normal equation in (c) to get a least square solution of $Ax = b$.

4. Find the equation $y = \beta_0 + \beta_1 x$ of the least–squares line that best fit the data points $(-1, 0)$, $(0, 1)$, $(1, 2)$, and $(2, 4)$.

5. A healthy child's systolic blood pressure p (in millimeters of mercury) and weight w (in pounds) are approximately related by the equation $p = \beta_0 + \beta_1 \ln w$, where $\ln w$ is the natural logorithm of w.

   (a) Use the experimental data

$$(w, p) = (44, 91), \ (61, 98), \ (81, 103), \ (113, 110), \ (131, 112)$$

to find the least–squares fit curve.

(b) Use part (a) to estimate the systolic pressure of a healthy child weighing 100 pounds.