Math 225 (Q1) Homework Assignment 10.

1. Let $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$.

(a) What is the rank of A?

(b) Find a singular value decomposition for A.

(c) Using part (b), find a basis of $\mathrm{Col}(A)$ and a basis of $\mathrm{Nul}(A)$.

2. Show that if A is a square matrix, then $|\det(A)|$ is the product of the singular values of A.

3. Let V be a vector space. Suppose S is a maximal set of linear independent vectors. That is, (i) the vectors in S are linearly independent and (ii) if we add one more vector (not from the set S) to the set S, then the resulting set (with one more vector than S) will no longer be linearly independent. Show that S is a basis of V.

4. Let $A = \begin{pmatrix} 4 & -2 \\ 2 & -1 \\ 0 & 0 \end{pmatrix}$.

(a) Find a SVD for A.

(b) Find the pseudoinverse, A^+, of A

5. Let V be a vector space. Suppose S is a minimal set of spanning vectors. That is, (i) the vectors in S span V (meaning, every vector in V is a linear combination of vectors in S) and (ii) if we remove one vector from the set S, then the resulting set (with one less vector than S) will no longer span V. Show that S is a basis of V.