TECHNION—ISRAEL INSTITUTE OF TECHNOLOGY
INSTITUTE OF ADVANCED STUDIES IN MATHEMATICS

INTERNATIONAL WORKSHOP

on

GEOMETRIC CONVEX ANALYSIS

will be held on

19-20 March, 2000

in

Room 814
Amado Mathematics Building
Technion, Haifa, Israel

Participants: Franck Barthe (France), Yoav Benyamini (Israel), Dario Cordero (France), Arye Dvoretzky (Israel), Effim Gluskin (Israel), Olivier Guédon (France), William B. Johnson (U.S.A.), Joram Lindenstrauss (Israel), Olga Maleva (Israel), Shahar Mendelson (Israel), Mathieu Meyer (France), Vitali Milman (Israel), Assaf Naor (Israel), Alain Pajor (France), Gideon Schechtman (Israel), Simeon Reich (Israel), Shlomo Reisner (Israel), David Saphar (Israel), Antonis Tsolomitis (Greece), Lior Tzafriri (Israel), Rafael Villa (Spain), Alexander Zaslavski (Israel), Mordecay Zippin (Israel), Artem Zvavitch (Israel)

Organizers: Yehoram Gordon and Alexander Litvak

For further information :
Sylvia Schur (Secretary)
Department of Mathematics
Technion-Israel Insitutute of Technology
32000 Haifa, Israel

iasm@techunix.technion.ac.il
Phone : 972 4 829 4278

For program details see:
http:/www.math.technion.ac.il/institute/convex-prog.html

Program

Sunday, 19 March
Morning session

10:00-12:00 Informal discussions

Afternoon session

14:00-14:50 Franck Barthe (Paris, France)
Isoperimetry for product measures, comparison with the Gaussian case

15:00-15:40 Rafael Villa (Seville, Spain)
Concentration of the distance for finite dimensional normed spaces

15:40-16:10 

16:10-16:40 Artem Zvavitch (Rehovot, Israel)
Embeddings of subspaces of L_p $(0<p<1)$ in ℓ_p^n via majorizing measures

16:50-17:30 Dario Cordero (Marne-la-Vallée, France)
Prekopa-Leindler inequalities in manifolds

18:00 Departure for dinner in Acre

Monday, 20 March
Morning session

09:00-09:30 Mordecay Zippin (Jerusalem, Israel)
A local minimality property of translation invariant projections on L/p(G)

09:40-10:20 Assaf Naor (Jerusalem, Israel)
Isomorphic embeddings of ℓ_p^m in ℓ_p^n

10:20-10:50 

10:50-11:30 Olivier Guédon (Paris, France)
Euclidean projections of a p-convex body

11:40-12:30 Antonis Tsolomitis (Samos, Greece)
John's theorem for an arbitrary pair of convex bodies

Afternoon session

14:30-15:10 Simeon Reich / Alexander Zaslavski (Haifa, Israel)
The set of divergent descent methods in a Banach space is sigma-porous

15:10-15:30 

Colloquium talk (Room 232)

15:30-16:25 Gideon Schechtman (Rehovot, Israel)
Affine approximation of Lipschitz maps between Banach spaces

Coffee will be available in the 8th floor faculty lounge

Abstracts

ISOPERIMETRY FOR PRODUCT MEASURES, COMPARISON WITH THE GAUSSIAN CASE

Franck Barthe
University of Marne-la-Vallée, France

We will prove some extremal properties of canonical half-spaces for product measures, in particular with respect to the isoperimetric problem and to the shift problem. We will always proceed by comparison with the Gaussian case and emphasize the formal similarities between several methods: Kanter's result on unimodality, Bobkov's functional approach, etc.

PRÉKOPA–LEINDLER INEQUALITY ON RIEMANNIAN MANIFOLDS

Dario Cordero-Erausquin
University of Marne-la-Vallée, France

This is joint work with R. J. McCann and M. Schmuckenschläger.

We derive inequalities on Riemannian manifolds that generalize inequalities of Prékopa–Leindler and Borell–Brascamp–Lieb obtained in the case of Euclidian space. The result sheds new light on this family of inequalities, the difference with the Euclidean case being a "distortion" factor that, roughly speaking, measures how far we are from the Euclidean space. The method relies on the mass transport on manifolds, recently obtained by McCann, together with computation on Jacobi Fields.

ISOMORPHIC EMBEDDINGS OF ℓ_p^m INTO ℓ_1^n

Assaf Naor
Hebrew University of Jerusalem

For $1 < p < 2$ and $\epsilon > 0$, what is the smallest C such that ℓ_p^n C-embeds into $\ell_1^{(1+\epsilon)n}$? It is conjectured by V. Milman and G. Schechtman that the answer is some $C(\epsilon)$ independent of n.

In this talk we will discuss joint work with A. Zvavitch which partially answers the problem by showing that we can take $C \leq (K \log n)^{1/q(1/\epsilon+1)}$ where $1/p + 1/q = 1$ and K is some universal constant.

THE SET OF DIVERGENT DESCENT METHODS IN A BANACH SPACE IS σ-POROUS

Simeon Reich and Alexander J. Zaslavski
Technion–Israel Institute of Technology

Given a Lipschitzian convex function f on a Banach space X, we consider a complete metric space $\mathcal{A}$ of vector fields V on X with the topology of uniform convergence on bounded subsets. With each such vector field we associate two iterative processes. We introduce the class of regular vector fields $V \in \mathcal{A}$ and prove that for a regular vector field V the values of the function f tend to its infimum for both processes. We then show that the complement of the set of regular vector fields is not only of the first category, but also σ-porous.

AFFINE APPROXIMATION OF LIPSCHITZ MAPS BETWEEN BANACH SPACES

Gideon Schechtman
Weizmann Institute of Science, Rehovot

Given a map f from a domain in a Banach space to another Banach space, and a point x in the domain, there are several ways to define a notion of linear approximation to f at x. The classical way is by (several different notions of) derivatives. Recently, several new definitions emerged and turned out to be useful in places where the classical notions do not suffice. We'll survey some of these notions, and their applications (to classification problems in non-linear functional analysis).

One such notion which is the subject of a recent study by Johnson, Lindenstrauss, Preiss and the lecturer is that of *ϵ-Fréchet differentiability*. A mapping f defined on a domain in a Banach space X into a Banach space Y is *ϵ-Fréchet differentiable* at x_0 if there is a bounded linear operator T from X to Y and a $\delta > 0$ so that

$$||f(x_0 + u) - f(x_0) - Tu|| < \epsilon||u|| \quad \text{for} \quad ||u|| \leq \delta.$$

We shall discuss the question as to which couples of Banach spaces have the property that every Lipschitz mapping from a domain in the first space to the second space has, for every $\epsilon > 0$, a point of ϵ-Fréchet differentiability.

JOHN'S THEOREM FOR AN ARBITRARY PAIR OF CONVEX BODIES

Antonis Tsolomitis
University of the Aegean, Samos

This is joint work with A. Giannopoulos and I. Perissinak.

We provide a generalization of John's representation of the identity for the maximal volume position of L inside K, where K and L are arbitrary smooth convex bodies in $\mathbb{R}^n$. The main Theorem is as follows:

Theorem . *Let L be of maximal volume in K. If $z \in \text{int}(L)$, we can find contact points $v_1, \ldots, v_m$ of $K - z$ and $L - z$, contact points $u_1, \ldots, u_m$ of the polar bodies $(K - z)^\circ$ and $(L - z)^\circ$, and positive reals $\lambda_1, \ldots, \lambda_m$, such that: $\sum \lambda_j u_j = o$, $\langle u_j, v_j \rangle = 1$, and*

$$Id = \sum_{j=1}^{m} \lambda_j u_j \otimes v_j .$$

It is also proved that z can be chosen in such a way that $\sum \lambda_j u_j = o = \sum \lambda_j v_j$, provided that L is a polytope and K has a C^2 boundary.

From this representation we obtain Banach-Mazur distance and volume ratio estimates.

CONCENTRATION OF THE DISTANCE IN FINITE DIMENSIONAL NORMED SPACES

Rafael Villa
University of Seville, Spain

Let $X = (\mathbb{R}^n, \|\cdot\|)$ be an n-dimensional normed space, $B = \{x \in \mathbb{R}^n : \|x\| \leq 1\}$ its closed unit ball and let σ be the Lebesgue measure restricted to B, normalized so that $\sigma(B) = 1$. Since $\sigma(rB) = r^n$, for large n, most of the points in B lie near its surface, so their norm is about 1. In this talk, our aim is to investigate the typical behaviour of the distance between *two* points in the ball.

This investigation is motivated by a number of results showing that in a wide variety of special spaces, it is possible to find many points, any two of which are roughly distance 1 apart. This problem is equivalent to the study of almost isometric embeddings of the ℓ_∞^N-cube in normed spaces. A volume argument easily shows that $N(\varepsilon) \leq \exp\{\phi(\varepsilon)n\}$ for some function ϕ, independent of X. On the other hand, some results on exponential estimates in particular spaces are known.

In the attempt to get a general strategy we deal with the distribution function of the random variable $\|x - y\|$ assuming x and y to be identically distributed independent random variables in the unit ball of X. This method allows us to get a common proof for a class of spaces containing those with a

EUCLIDEAN PROJECTIONS OF A p-CONVEX BODY

Olivier Guedon
Paris, France

This is joint work with A.E. Litvak.

Given a p-convex body K in $\mathbb{R}^n$, is it possible to find a "good" projection P of rank k, i.e., such that PK is convex up to a constant depending only on p ?

I will present some results about Euclidean projections of K, i.e., given integer k between $\ln n$ and $n/2$, what is the best estimate of $\inf\{d_{PK}, P \text{ projection of rank } k\}$, where d_{PK} denotes the Banach Mazur distance between PK and the Euclidean ball? The method can also be used to study the distance between PK and its convex hull.

The sharpness of the result is proved by volumic consideration. This is obtained proving precise estimates of entropy numbers of identity operator acting between ℓ_p and ℓ_r spaces for the case $0 < p < r \leq \infty$.