Limits of special random walks in polygons and a class of stochastic fractals

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Abstract

We consider a special random walk of a particle in a polygon and obtain, as a limit, cut off fractals in the polygon, which are described in terms of uniform distributions of probability on the corresponding fractals. Serpinski triangle is an example. Changing a parameter of the random walk we obtain in a limit fractal type distributions with a tight support on the polygon. Construction for the limit distributions forming is essentially based on a generalization of the Fibonacci numbers.

Refs

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Definition of Pure type distribution

A random variable X has a pure type distribution, if exactly one condition from the following 3 conditions takes place:

1. There erxist finit or countable set D such that $P(X \in D) = 1$.
2. For every $x \in \mathbb{R}$ it holds $\mathbb{P}(X = x) = 0$, but there exists a Borel D such that $P((X \in D) = 1)$ with the Lebesgue measure $\mu(D) = 0$.
3. $P(X \in \mathrm{d}x) \prec \mu(\mathrm{d}x)$.

Jessen–Wintner's Theorem on Pure type (1935)

Let $X_1, X_2, \ldots$ — be i.i.d. rv's such that:

1. $\sum_1^n X_k \to X$ a.s. as $n \to \infty$;
2. For every $k \in \mathbb{N}$ there exists a countable set F_k: $P(X_k \in F_k) = 1$.

Then distribution of X has the Pure type.

Polski zloty real data and simulation

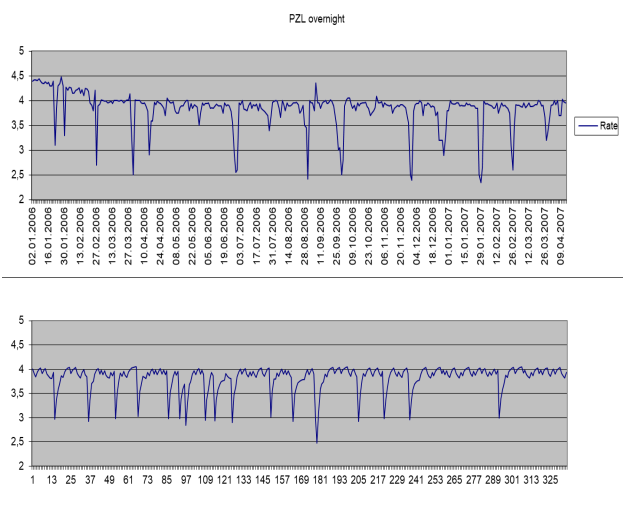

Figure: $\alpha = 1/2$, 3 attractors: 2.0 (prob =0.05), 3.79 (prob =0.42), and 4.05 (prob =0.53)