Alberta High School Mathematics Competition

Part II February 6, 2008.

Problem 1.
The function $f(n) = an + b$, where a and b are integers, is such that for every integer n, $f(3n+1)$, $f(3n)+1$ and $3f(n)+1$ are three consecutive integers in some order. Determine all such $f(n)$.

Problem 2.
In a contest, no student solved all problems. Each problem was solved by exactly three students and each pair of problems was solved by exactly one student. What is the maximum number of problems in this contest?

Problem 3.
From a jar of candies, Autumn takes $a\%$ of the candies plus a more candies. Brooke takes $b\%$ of the remaining candies plus b more candies. Here, a and b are positive integers less than 100. If Autumn and Brooke have taken the same number of candies, determine all possible values of a and b.

Problem 4.
Let a, b, c and d be real numbers. The equation $x^2 + ax + b = 0$ has two real roots. On the other hand, the equation $(x^2 - 2cx + d)^2 + a(x^2 - 2cx + d) + b = 0$ has no real roots. Prove that $d^2 + ad + b > c^4$.

Problem 5.
In triangle ABC, $AB = AC$ and $\angle A = 100^\circ$. D is the point on BC such that $AC = DC$, and F is the point on AB such that DF is parallel to AC. Determine $\angle DCF$.