Alberta High School Mathematics Competition

Second Round February 7, 2007.

Problem 1.
Determine all positive integers n such that n is divisible by any positive integer m which satisfies $m^2 + 4 \leq n$.

Problem 2.
The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 are arranged to form a 5×3 table in each of the 15! possible ways. For each table, we compute the sum of the three numbers in each row, and record in a list the largest and the smallest of these sums. Determine the sum of the $2 \times 15!$ numbers on our list.

Problem 3.
One angle of a triangle is $36°$ while each of the other two angles is also an integral number of degrees. The triangle can be divided into two isosceles triangles by a straight cut. Determine all possible values of the largest angle of this triangle.

Problem 4.
Let a, b and c be distinct non-zero real numbers such that $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}$. Determine all possible values of $a^3 + b^3 + c^3$.

Problem 5.
A survey in Alberta was sent to some teachers and students, a total of $2006 = 2 \times 17 \times 59$ people. Exactly $a\%$ of the teachers and exactly $b\%$ of the students responded, yielding an overall response rate of exactly $c\%$, where a, b and c are integers satisfying $0 < a < c < b < 100$. For each possible combination of values of a, b and c, determine the total number of teachers and the total number of students who responded to the survey.