The Alberta High School Mathematics Competition

Part I, November 21, 2012

1. Each day, Mr. Sod visited pubs A, B, C and D in that order, always spending \$35, \$12, \$40 and \$27 at the respective places. His total expenditure at the pubs, from the beginning of the month up to a certain moment that month, was \$1061. Which pub would he be visiting next?

   (a) A (b) B (c) C (d) D (e) impossible total

2. Meeny, Miny and Moe were playing tennis. From the second game on, the one who sat out the preceding game would replace the loser of that game. At the end, Meeny played 17 games and Miny played 35 games. How many games did Moe play?

   (a) 18 (b) 26 (c) 36 (d) 52

   (e) not uniquely determined

3. A circle of diameter r is drawn inside a circle of diameter R. For which of the following pairs (r, R) is the area of the smaller circle closest to half the area of the larger circle?

   (a) (1, 3) (b) (2, 4) (c) (3, 5) (d) (4, 6) (e) (5, 7)

4. A quadratic polynomial $f(x)$ satisfies $f(0) = 1,\ f(1) = 0$ and $f(2) = 3$. What is the value of $f(3)$?

   (a) -3 (b) 1 (c) 2 (d) 10 (e) none of these

5. $ABCD$ is a square. E and F are points on the segment BC such that $BE = EF = FC = 4$ cm. The segments AF and DE intersect at G. What, in cm^2, is the area of triangle EFG?

   (a) 6 (b) $4\sqrt{3}$ (c) 8 (d) 12 (e) none of these

6. For how many integers $n \geq 2$ is the sum of the first n positive integers a prime number?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

7. In a test, Karla solved four-fifth of the problems and Klaus solved 35 problems. Half of the problems were solved by both of them. The number of problems solved by neither was a positive one-digit number. What was this number?

   (a) 1 or 2 (b) 3 or 4 (c) 5 or 6 (d) 7 or 8 (e) 9

8. What is the largest possible integer a such that exactly three of the following statements are true: $a < 1$, $a > 2$, $a < 3$, $a > 4$ and $a < 5$?

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

9. A rectangle with integer length and width in cm has area 70 cm^2. Which of the following, in cm, cannot be the length of the perimeter of the rectangle?

(a) 34 (b) 38 (c) 74 (d) 98 (e) 142

10. The positive integer n is such that between $n^2 + 1$ and $2n^2$ there are exactly five different perfect squares. How many such n can we find?

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

11. $ABCD$ is a rectangle such that $AD - AB = 15$ cm. $PQRS$ is a square inside $ABCD$ whose sides are parallel to those of the rectangle, with P closest to A and Q closest to B. The total area of $APSD$ and $BQRC$ is 363 cm^2 while the total area of $APQB$ and $CRSD$ is 1113 cm^2. What, in cm^2, is the area of $PQRS$?

(a) 900 (b) 1600 (c) 2500 (d) 3600

(e) not uniquely determined

12. Weifeng writes down 28 consecutive numbers. If both the smallest and the largest number are perfect squares, what is the smallest number she writes down ?

(a) 9 (b) 36 (c) 100 (d) 169

(e) not uniquely determined

13. If the positive numbers a and b satisfy $\dfrac{1}{a^2 + 4b + 4} + \dfrac{1}{b^2 + 4a + 4} = \dfrac{1}{8}$, what is the maximum value of $a + b$?

(a) $\frac{3}{2}$ (b) 2 (c) $\frac{5}{2}$ (d) 4 (e) none of these

14. The incircle of triangle ABC is tangent to AB and AC at F and E respectively. If $BC = 1$, $\angle A = 90°$ and $\angle B \neq \angle C$, what is the distance from the midpoint of BC to EF?

(a) $\frac{\sqrt{2}}{4}$ (b) $\frac{\sqrt{2}}{2}$ (c) $\frac{3\sqrt{2}}{4}$ (d) $\sqrt{2}$

(e) not uniquely determined

15. At the beginning of the year, there were more robots than androids. On the first day of each month, each robot made 7 androids and each android made 7 robots. The next day, each old android would pick a fight with a new android, and they would destroy each other. At the end of the year, there were 46875 million robots and 15625 million androids. What was the difference between the numbers of robots and androids at the beginning of the year?

(a) less than 10 (b) at least 10 but less than 100 (c) at least 100 but less than 1000

(d) at least 1000 but less than 10000 (e) at least 10000

16. Let m and n be positive integers such that 11 divides $m + 13n$ and 13 divides $m + 11n$. What is the minimum value of $m + n$?

(a) 24 (b) 26 (c) 28 (d) 30 (e) 34