The Alberta High School Mathematics Competition

Part I, November 15, 2011

1. If $2^{2012} + 4^{1006} = 2^n$, then n is

   (a) 2013 (b) 2014 (c) 3018 (d) 4024 (e) not an integer

2. Mini-marshmallows are cubes 1 centimetre on each side, while giant marshmallows are cubes 3 centimetres on each side. The number of mini-marshmallows whose combined surface area is the same as the surface area of one giant marshmallow is

   (a) 3 (b) 6 (c) 9 (d) 27 (e) 54

3. The number of customers in a restaurant on Tuesday is 20% more than the number on Monday, the number of customers on Wednesday is 50% more than the number on Monday, and the number of customers on Wednesday is n% more than the number on Tuesday. The value of n is

   (a) 20 (b) 25 (c) 30 (d) 50 (e) none of these

4. Sawa starts from point S and walks 1 kilometre north, 2 kilometres east, 3 kilometres south and 4 kilometres west. At this point, her distance, in kilometres, from S is

   (a) $\sqrt{5}$ (b) $2\sqrt{2}$ (c) 4 (d) 8 (e) 10

5. A millenium number is a positive integer such that the product of its digits is 1000. The number of six-digit millenium numbers is

   (a) 60 (b) 120 (c) 140 (d) 180 (e) 200

6. Let $x_1, x_2, \ldots$ be a sequence of positive rational numbers such that $x_1 = 16$, $x_2 = 32$ and $x_n = \frac{x_{n-l}+x_{n-2}}{2}$ for all positive integers $n \geq 3$. Then the value of x_6 is

   (a) 24 (b) 25 (c) 26 (d) 27 (e) 28

7. The number of real solutions of the equation $2x^2 - 2x = 2x\sqrt{x^2 - 2x} + 1$ is

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

8. Let $f(x)$ be a quadratic polynomial. If $f(1) = 2$, $f(2) = 4$ and $f(3) = 8$, then the value of $f(4)$ is

   (a) 12 (b) 14 (c) 15 (d) 16 (e) 18

9. A lucky number is a positive integer n such that 7 is the largest divisor of n which is less than n. The number of lucky numbers is

(a) 1 (b) 2 (c) 3 (d) 4 (e) more than 4

10. The perimeter of a square lawn consists of four straight paths. Annabel and Bethany started at the same corner at the same time, running clockwise at constant speeds of 12 and 10 kilometres per hour respectively. Annabel finished one lap around the lawn in 1 minute. During this minute, the number of seconds that Annabel and Bethany were on the same path is

(a) 36 (b) 42 (c) 48 (d) 50 (e) none of these

11. $ABCD$ is a quadrilateral with $AD = BC$ and AB parallel to DC. It is only given that the lengths of AB and DC are 20 and 15 centimetres respectively. Adrian puts n copies of this tile together so that the edge BC of each copy coincides with the edge AD of the next, and the edges DC of all copies together form a regular n-sided polygon. Then the value of n is

(a) 6 (b) 8 (c) 12 (d) 20 (e) not uniquely determined

12. The sum of twenty positive integers, not necessarily different, is 462. The largest possible value of greatest common divisor of these numbers is

(a) 21 (b) 22 (c) 23 (d) 33 (e) 42

13. The largest real number m such that $(x^2 + y^2)^3 > m(x^3 + y^3)^2$ for any positive real numbers x and y is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) none of these

14. Of the 49 squares of a 7×7 square sheet of paper, two are to be coloured black while the others remain white. Two colourings are called distinct if one cannot be obtained from the other by rotating the sheet of paper about its centre. The number of distinct colourings is

(a) 288 (b) 294 (c) 296 (d) 300 (e) 588

15. The lengths of the sides of triartgle ABC are consecutive positive integers. D is the midpoint of BC and AD is perpendicular to the bisector of $\angle C$. The product of the lengths of the three sides is

(a) 24 (b) 60 (c) 120 (d) 210 (e) 336

16. For any real number r, $\lfloor r \rfloor$ is the largest integer less than or equal to r. For example, $\lfloor \pi \rfloor = 3$. Let n be a positive integer. Let $a_1 = n$, $a_2 = \lfloor \frac{a_1}{3} \rfloor$, $a_3 = \lfloor \frac{a_2}{3} \rfloor$ and $a_4 = \lfloor \frac{a_3}{3} \rfloor$. The number of positive integers n from 1 to 1000 inclusive such that none of a_1, a_2, a_3 and a_4 is divisible by 3 is

(a) 144 (b) 192 (c) 210 (d) 280 (e) none of these