The Alberta High School Mathematics Competition
Part I, November 17, 2009

1. If $2^x = 3^y$, then 4^x is equal to

   (a) 5^y (b) 6^y (c) 8^y (d) 9^y (e) none of these

2. Caroline bought some bones for her 7 dogs. Had she owned 8 dogs, she could have given each the same number of bones. As it was, she needed two more bones to give each dog the same number of bones. The number of bones she could have bought was

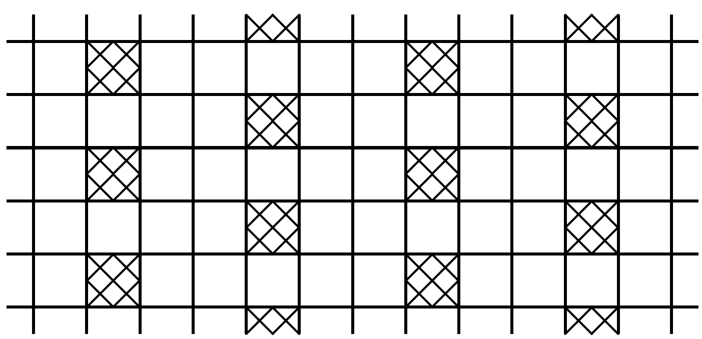
   (a) 16 (b) 24 (c) 32 (d) 40 (e) 48

3. Ace calculates the average of all the integers from 1 to 100. Bea calculates the average of all the integers from 1001 to 1100 and subtracts 1000. Cec calculates the average of all the integers from 1000001 to 1000100 and subtracts 1000000. The largest answer is given by

   (a) Ace only (b) Bea only (c) Cec only (d) exactly two of them
   (e) all three of them

4. A large rectangular gymnasium floor is covered with unit square tiles, most of them blank, in the pattern shown in the diagram below. Of the following fractions, the one nearest to the fraction of tiles which are not blank is

   (a) $\frac{1}{12}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$ (e) $\frac{1}{4}$



5. The number of integers between 1 and 2009 inclusive which can be expressed as the difference of the squares of two integers is

   (a) 1 (b) 502 (c) 1005 (d) 1507 (e) 2009

6. Among the positive integers with six digits in their base-10 representation, the number of those whose digits are strictly increasing from left to right is

   (a) between 1 and 50 (b) between 51 and 100 (c) between 101 and 500

   (d) between 501 and 1000 (e) greater than 1000

7. The number of arrangements of the letters AABBCC in a row such that no two identical letters are adjacent is

(a) 30 (b) 36 (c) 42 (d) 48 (e) none of these

8. If 2^{2009} has m digits and 5^{2009} has n digits in their base-10 representations, then the value of $m + n$ is

(a) 2007 (b) 2008 (c) 2009 (d) 2010 (e) 2011

9. An equilateral triangle has area $2\sqrt{3}$. From the midpoint of each side, perpendiculars are dropped to the other two sides. The area of the hexagon formed by these six lines is

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\sqrt{3}$ (d) 2 (e) none of these

10. Two sides of an *obtuse* triangle of positive area are of length 5 and 11. The number of possible integer lengths of the third side is

(a) 3 (b) 4 (c) 6 (d) 8 (e) 9

11. $Q(x)$ is a polynomial with integer coefficients such that $Q(9) = 2009$. If p is a prime number such that $Q(p) = 392$, then p can

(a) only be 2 (b) only be 3 (c) only be 5
(d) only be 7 (e) be any of 2, 3, 5 and 7

12. A parallelogram has two opposite sides 5 centimetres apart and the other two opposite sides 8 centimetres apart. Then the area, in square centimetres, of the parallelogram

(a) must be at most 40 and can be any positive value at most 40
(b) must be at least 40 and can be any value at least 40
(c) must be 40 (d) can be any positive value (e) none of these

13. The number of positive integers n such that $\sqrt{n + \sqrt{n + \cdots + \sqrt{n}}} < 10$ for any finite number of square root signs is

(a) 10 (b) 90 (c) 91 (d) 99 (e) 100

14. A chord of a circle divides the circle into two parts such that the squares inscribed in the two parts have areas 16 and 144 square centimetres. In centimetres, the radius of the circle is

(a) $2\sqrt{10}$ (b) $6\sqrt{2}$ (c) 9 (d) $\sqrt{85}$ (e) 10

15. The number of prime numbers p such that $2^p + p^2$ is also a prime number is

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

16. Suppose that $2 - \sqrt{99}$ is a root of $x^2 + ax + b$ where b is a negative real number and a is an integer. The largest possible value of a is

(a) -4 (b) 4 (c) 7 (d) 8 (e) none of these