Alberta High School Mathematics Competition

First Round November 21, 2006.

- 1. The value of $2^4 4^8 8^{16}$ is
 - (a) 2^{16} (b) 2^{52} (c) 2^{68} (d) 2^{84} (e) none of these

2. The number of non-congruent rectangles with integer sides and area $2006 = 2 \times 17 \times 59$ is

(a) 3 (b) 4 (c) 6 (d) 8 (e) none of these

3. The number of pairs (m, n) of positive integers such that $m^4 + n = 100000001$ is

(a) 100 (b) 101 (c) 200 (d) 201 (e) none of these

4. In a city, all streets run north-south or east-west, dividing the city into squares. A, B, C and D are four students who live at four street intersections which define a rectangle, with A and C at opposite corners of this rectangle. They all go to the same school, which is at some street intersection within this rectangle. Each goes to school by a most direct route along streets. A travels 10 blocks, B 20 blocks and C 50 blocks. The number of blocks D travels is

- (a) 10 (b) 20 (c) 30 (d) 40 (e) 50
- 5. When $(1+x)(1+x^2)(1+x^3)(1+x^4)(1+x^5)$ is expanded, the coefficient of the term x^9 is
 - (a) 0 (b) 1 (c) 2 (d) 3 (e) 4
- 6. On a plane are two points A and B at a distance 5 apart. The number of straight lines in this plane which are at a distance 2 from A and at a distance 3 from B is
 - (a) 0 (b) 1 (c) 2 (d) 3 (e) 4
- 7. A country has three provinces, each province has three cities, each city has three wards and each ward has three electors. In a two-way election, a candidate wins a ward by getting more votes in the ward, wins a city by winning more wards in the city, wins a province by winning more cities in the province, and wins the election by winning more provinces. Only electors may vote, and they must vote. The minimum number of votes needed to guarantee winning the election is
 - (a) 41 (b) 54 (c) 66 (d) 81 (e) none of these

8. The teacher asked, "What is the largest possible diameter of a circular coin of negligible thickness which may be stored in a rectangular box with inner dimensions $7 \times 8 \times 9$?" Ace said less than 8, Bea said 8, Cec said strictly between 8 and 9, Dee said 9 and Eve said more than 9. The one who was right was

(a) Ace (b) Bea (c) Cec (d) Dee (e) Eve

9. If a and b are two real numbers which satisfy a + b - ab = 1 and a is not an integer, then b

(a) is never an integer

- (b) must be some positive integer
- (c) must be some negative integer (d) must be equal to 0
- (e) may either be an integer or a non-integer

10. The non-zero numbers a, b, c, x, y and z are such that $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$. The value of $\frac{xyz(a+b)(b+c)(c+a)}{abc(x+y)(y+z)(z+x)}$ is

(a) 0 (b) $\frac{1}{3}$ (c) 1 (d) 3

(e) dependent on the common value of $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

11. If k is a rational number such that $\sqrt[3]{9\sqrt{3} - 11\sqrt{2}} = \sqrt{3} + k\sqrt{2}$, then the value of k is (a) -2 (b) -1 (c) 1 (d) 2 (e) none of these

- 12. All positive integers which can be expressed as a sum of one or more different integer powers of 5 are written in increasing order. The first three terms of this sequence are 1, 5 and 6. The fiftieth term is
 - (a) 3751 (b) 3755 (c) 3756 (d) 3760 (e) 3761
- 13. Consider all polynomials whose coefficients are all integers, whose roots include $\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{2}}{3}$ and whose degree is as small as possible. Among the coefficients of these polynomials, the smallest positive coefficient is
 - (a) 1 (b) 6 (c) 35 (d) 36 (e) none of these
- 14. Colleen used a calculator to compute $\frac{a+b}{c}$, where a, b and c are positive integers. She pressed a, +, b, /, c and = in that order, and got the answer 11. When she pressed b, +, a, /, c and = in that order, she was surprised to get a different answer 14. Then she realized that the calculator performed the division before the addition. So she pressed (, a, +, b,), /, c and = in that order. She finally got the correct answer, which is
 - (a) 4 (b) 5 (c) 20 (d) 25 (e) none of these
- 15. The base of a tetrahedron is an equilateral triangle of side 1. The fourth vertex is at a distance 1 above the centre of the base. The radius of the sphere which passes through all four vertices of the tetrahedron is
 - (a) $\frac{1}{3}$ (b) $\frac{\sqrt{3}}{6}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{3}$ (e) $\frac{2}{3}$
- 16. If p(x) is a polynomial of degree 4 such that p(-1) = p(1) = 5 and p(-2) = p(0) = p(2) = 2, then the maximum value of p(x) is
 - (a) 5 (b) 6 (c) 7 (d) 10 (e) none of these