The Alberta High School Mathematics Competition
Part I, 2015

Question 1.
How many three digit numbers have the product of the three digits equal to 5?

(a) 1 (b) 2 (c) 3 (d) 5 (e) 6

Question 2.
Let m, n be positive integers such that $2^{30}3^{30} = 8^m 9^n$. Determine the value of $m + n$.

(a) 15 (b) 20 (c) 25 (d) 30 (e) 35

Question 3.
The x-intercept, y-intercept, and slope of a certain straight line are three nonzero real numbers. The number of *negative* numbers among these three numbers is:

(a) 0 or1 (b) 1 or 2 (c) 2 or 3 (d) 0 or 2 (e) 1 or 3

Question 4.
The length of a certain rectangle is increased by 20% and its width is increased by 30%. Then its area is increased by:

(a) 25% (b) 48% (c) 50% (d) 56% (e) 60%

Question 5.
Each of Alan, Bailey, Clara and Diane has a number of candies. Compared with the average of the number of candies each person has, Alan has 6 more than the average, Bailey has 2 more than the average, Clara has 10 fewer than the average and Diane has k candies more than the average. Determine k.

(a) 1 (b) 2 (c) 3 (d) 4 (e) not uniquely determined

Question 6.
Ellie wishes to choose three of the seven days (Monday, Tuesday,..., Sunday) on which to wash her hair every week, so that she will never wash her hair on consecutive days. The number of ways she can choose these three days is:

(a) 6 (b) 7 (c) 8 (d) 10 (e) 14

Question 7.
How many different sets of two or more consecutive whole numbers have sum 55?

(a) 2 (b) 3 (c) 4 (d) 5 (e) none of these

Question 8.
There are 5 boys and 6 girls in a class. A committee of three students is to be made such that there is a boy and a girl on the committee. In how many different ways can the committee be selected?

(a) 100 (b) 135 (c) 145 (d) 155 (e) 165

Question 9.
In a class with 20 students, 14 wear glasses, 15 wear braces, 17 wear ear-rings and 18 wear wigs. What is the minimum number of students in this class who wear all four items?

(a) 4 (b) 6 (c) 7 (d) 9 (e) 10

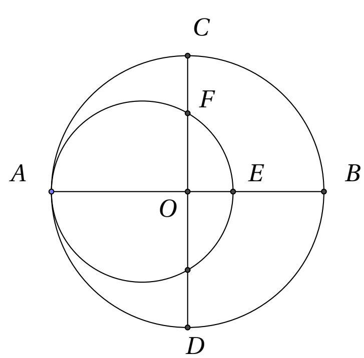
Question 10.
Each person has two legs. Some are sitting on three-legged stools while the others are sitting on four-legged chairs such that all the stools and chairs are occupied. If the total number of legs is 39, how many people are there?

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

Question 11.
The number of integers n for which the fraction $\frac{2^{2015}}{5n+1}$ is an integer is

(a) 503 (b) 504 (c) 1006 (d) 1007 (e) 1008

Question 12.
In the diagram below, which is not drawn to scale, the circles are tangent at A, the centre of the larger circle is at O and the lines AB and CD are perpendicular.



If $EB = 3$ and $FC = 2$ then the radius of the smaller circle is

(a) 4/3 (b) 5/3 (c) 5/2 (d) 3 (e) 7/2

Question 13.
Consider the expansion

$$\left(1+x+x^2+\cdots+x^{50}\right)^3 = c_0+c_1x+c_2x^2+\cdots+c_{150}x^{150}.$$

The value of the coefficient c_{50} is

(a) 1274 (b) 1275 (c) 1326 (d) 1378 (e) none of these

Question 14.
A 1000 digit number has the property that every two consecutive digits form a number that is a product of four prime numbers. The digit in the 500th position is

(a) 2 (b) 4 (c) 5 (d) 6 (e) 8

Question 15
Points E and F are on the sides BC and respectively CD of the parallelogram $ABCD$ such that $\frac{EB}{EC} = \frac{2}{3}$ and $\frac{FC}{FD} = \frac{1}{4}$. Let M be the intersection of AE and BF. The value of $\frac{AM}{ME}$ is equal to

(a) 11 (b) $11\frac{1}{2}$ (c) 12 (d) $12\frac{1}{2}$ (e) $12\frac{3}{4}$

Question 16.
Each of Alvin, Bob and Carmen spent five consecutive hours composing problems. Alvin started alone, and was later joined by Bob. Carmen joined in before Alvin stopped. When one person was working alone 4 problems were composed per hour. When two people were working together, each only composed 3 problems per hour. When all three were working, each composed only 2 problems per hour. No coming or going occurs during the composition of any problem. At the end, 46 problems were composed. How many were composed by Bob?

(a) 14 (b) 15 (c) 16 (d) 17 (e) 18